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side of the board and the two front legs, and two other braces as an isosceles triangle to the third, or back leg, and then connecting the legs together, the connexion between the front legs being rigid, I made a very firm stand or table, and with the unexpected result, that whereas, to use engineering terms, the first six braces, viz. the four between the sides of the board and the legs, and the two connecting the two front legs to the third or back leg, act as ties, the seventh or rigid connexion between the two front legs becomes a strut, and, as such, its action is to tighten all the other ties or braces, thus giving the table the requisite degree of stability.

The braces I have in this case made of spring steel for lightness, but they may be of chain or wire, and if with the strut were made adjustable, by screws or otherwise, the table could be adjusted as to level, or to any required degree of obliquity.

As to the different uses to which a stand of this kind is applicable, I may mention the old-fashioned but useful surveying instrument, the Plane Table, which, with this mode of bracing, may be made of much larger size than is usual, and be constructed with greater accuracy. It would also form a firm and convenient stand for various uses in the observatory as well as in the field; light, portable, and easily taken asunder and stowed in a small space, at the same time not interfering with the other uses of the tripod. Doubtless other uses will suggest themselves in practice.

Memoir on the Theories of the four Superior Planets, Jupiter, Saturn, Uranus, and Neptune. By M. Le Verrier.

An abstract of a memoir by M. Le Verrier on the theories of the four exterior planets is inserted in No. xxi., for May 20, 1872, of the *Comptes Rendus*. The subject is of great importance, and one deeply interesting to many Fellows of this Society. The following is a translation of the principal portion of the abstract.

"On several occasions I have had the honour of laying before the Academy a series of investigations relating to the system of the four planets nearest to the Sun; viz., *Mercury*, *Venus*, the *Earth*, and *Mars*. Although at an earlier period, I had already been occupied in the study of the larger planets, I have felt the necessity, before continuing the investigation, of establishing on a solid foundation the theory of the *Earth's* motion, which serves as the basis of all the others. This study led me to the consideration of the three planets situated nearest the *Earth*, and which constitute the inferior portion of the planetary system.

"These researches have shown that the motions of the *Earth* and *Venus* are represented by the theory with all the accuracy which the observations will admit of. *Mercury* and *Mars*, however, showed some irregularities.

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"The difficulties which the last two planets presented to us have in a great measure disappeared after a careful examination of the subject. In order to establish an agreement between the theoretical and observed results, we have come to the conclusion that it would suffice if we were to increase the secular motions of the perihelia of *Mercury* and *Mars*, which would imply the existence in the neighbourhood, both of *Mercury* and *Mars*, of sensible quantities of matter of which we hitherto had no knowledge.

"With regard to *Mars*, these deductions have been confirmed. It has been proved that the matter, of which no account had hitherto been taken, ought to have been added to the *Earth* itself; the estimate of the mass of our planet being $\frac{1}{8}$th too small.

"With regard to *Mercury*, this confirmation is not yet complete. Several astronomers have placed on record the passage over the *Sun's* disk of certain small bodies, which could be nothing else but very small planets, but it has not been possible to determine the orbit of any one of them. Whether we have to deal with the action of a certain number of small masses, or with that of matter distributed in the neighbourhood of the *Sun*, the theory of *Mercury* has been determined with considerable care, and the transits of the planet over the *Sun's* disk furnish us with observations too precise to admit of any doubt of the accuracy of our results; especially as they have been obtained in the same manner as for *Mars*, and for this latter planet the confirmation which the theory has received leaves nothing more to be desired.

"Similar researches into the system of the four large planets farthest from the *Sun* would be interesting, as they would furnish us with data concerning any matter, still unknown to us, that may be situated in those regions. It is not impossible that, by these means, we might succeed in proving the existence of some planet beyond *Neptune*, and in limiting the space in which a search for such an object may be confined. At any rate we should have collected the materials necessary for the more speedy accomplishment of future discoveries.

"The first part of the work to be undertaken consists in the determination of the disturbances which each of the four planets experiences from the action of the others. It is this first part that I present to the Academy to-day.

"The respective theories of *Jupiter* and *Saturn* are not free from difficulty on account of the magnitude of the masses of these planets, and because the ratio existing between the mean motions of *Jupiter* and *Saturn* is very nearly represented by 5 to 2. Under these circumstances, some of the terms rise to a very considerable value, which requires the approximations to be carried farther. Besides which, the series of multiples of the angles, by following which the perturbing functions and perturbations are developed, have to be carried very far to avoid neglecting any term of the longitude exceeding one hundredth of a second, as we resolved to do. Again, the terms depending on the second order

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in relation to the perturbing masses, become very sensible, and their determination is a laborious operation.

"Two methods may be adopted for the determination of the inequalities of the first and second order in terms of the masses.

"The method of *interpolation* has this advantage; the actual work of the greater part of the investigation may be given to a computer; the numerous checks which present themselves preventing the possibility of any serious error escaping notice. On the other hand, this method has this inconvenience, that the computation being possible only for numerical data, everything has to be recommenced as soon as these data have undergone any considerable change depending upon the secular variations of the elements of the orbits.

"The development of the perturbing functions, as well as that of the inequalities of the elements under an algebraic form, in which the terms which vary with the time are left indeterminate, leads, on the contrary, to expressions which will serve for all succeeding ages. This is the course which it appeared to me necessary to follow in order to render a really important service to astronomy.

"Consequently, in the expressions I have adopted, the excentricities, the inclinations, and the longitudes of the perihelia and nodes, are, like the mean longitude, left in the form of variable quantities. The mean portions of the major axes are alone treated as given numbers, as is quite allowable, since these undergo no secular variation.

"The first chapter is devoted to the development of the transcendentals on which the determination of the coefficients depends.

"In the second chapter we give the algebraic development of all the terms of the perturbing functions which occur in the theory of *Jupiter* and *Saturn*.

"The terms of the great inequality depending on five times the mean motion of *Saturn*, less twice that of *Jupiter*, are developed with care to the seventh order.

"The third chapter contains the numerical expressions peculiar to *Jupiter* and *Saturn*.

"In the fourth chapter the perturbations of *Jupiter* are determined to the first order in relation to the mass of *Saturn*. For example, for the part of the great inequality depending on the angle

$$5\,l' - 2\,\lambda - \varpi' - 2\,\omega$$

the following expression is found:—

$$\delta\,l = (1 + \mu')\left(\begin{matrix} 791''\cdot32\,\beta^2\beta' - 2''\cdot00\,\beta^4\beta' - 15''\cdot06\,\beta^2\beta'^3 - 1''\cdot25\,\beta^2\beta'\,\varkappa^2 \\ +\,0''\cdot07\,\beta^4\beta'^3 + 0''\cdot12\,\beta^2\beta'^5 \end{matrix}\right)\text{sin.}\left(\begin{matrix} 5\,l' - 2\,\lambda \\ -\varpi' - 2\,\omega \end{matrix}\right)$$

in which $1 + \mu'$ represents the real mass of *Saturn*; β and β' are the ratios of the excentricities of *Jupiter* and *Saturn* at any

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given epoch, to their values at the fundamental epoch fixed at mean noon on January 1, 1850; $\varkappa$ is the ratio of the mutual inclination of the orbits of *Jupiter* and *Saturn* to the value of this inclination at the fundamental epoch; π' and ω represent the longitudes of the perihelia of *Saturn* and *Jupiter*.

"The fifth chapter contains the inequalities of the elements of *Saturn* in the same form.

"The inequalities of the second order in terms of the masses are sufficiently sensible to make a careful determination of them important; the fifth chapter is devoted to this object. It is necessary to put them in the same form as for those of the first order, so that by combining the whole of them we may have the complete expressions for the periodical variations of the elements in the form in which they may be easily referred to any given epoch.

"On account of the magnitude of the perturbations of the first order, and the smallness of the divisors $2\,n' - n$, $3\,n' - n$, $5\,n' - 2\,n$, and $10\,n' - 4\,n$, a large number of terms of the second order are sensible; they depend on the combination of different groups, and in each of these groups there are a considerable number of terms of which notice must be taken, if we wish to obtain an accurate result.

"For example, the term depending on the angle $5\,l' - 2\,\lambda - 2\,\omega - \pi'$, corresponding to that which we have already given to the first order, depends on not less than 18 combinations of groups, taken two and two, in each of which about 10 combinations of terms occur. The total gives an angle of 17 seconds for the co-efficient.

"The determination of the complete expression of these terms has been brought into a form, which, though leaving it complicated, is one in which we may proceed securely, leaving the variable elements indeterminate as for the first order.

"But how can we feel certain that in the course of such numerous operations no error has been committed?

"In the algebraic form, by attributing to the two planets determinate mean longitudes; for example, by placing the planets at their perihelia or their aphelia, we can obtain simple expressions of the corresponding value of the perturbing function, and thus deduce relations between the co-efficients of the general expression of this function, which form a valuable means of verification. These verifications, carefully made *à posteriori*, have been all found exact. The only exception has been for a very small term of the perturbing function of *Saturn*, arising from the reaction of *Jupiter* upon the *Sun*; a term of a very high order, quite insensible, which had changed its sign.

"In the numerical form, although all the co-efficients of the developments have been three times revised, we wished to determine their sums anew by the method of interpolation, of which we have already spoken; and we have even continued the work far enough to form an independent determination of the question

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we are discussing. This second investigation is in complete accordance with the first.

"We may then hope that astronomers will accept with confidence the work which we now place before them. It is our intention to print elsewhere all the details necessary to enable any person to verify the parts which he may be interested in examining more closely. In this manner, should a doubt arise on any point, it will easily be seen in what direction the work of verification should be directed.

"The two succeeding sections comprise the theories of the perturbations of *Jupiter* by *Uranus* and *Neptune*.

"The fourth contains the perturbations of *Saturn* by *Uranus* and *Neptune*.

"Then follow three sections, containing the perturbations of *Uranus* by *Jupiter*, *Saturn*, and *Neptune*. The investigation of the perturbations of *Uranus* by *Saturn* was made many years ago, but I then followed the method of interpolation. In the present work, in which the results agree with the former one, I have adopted the algebraic form, which can be applied to any epoch.

"The last sections contain the perturbations of *Neptune*, produced by *Jupiter*, *Saturn*, and *Uranus*.

"To bring this work to a satisfactory conclusion, it will be necessary,—

"1. To calculate the formulæ, and to reduce them into the form of provisional tables.

"2. To combine all the accurate observations of the four planets, and to re-discuss them in order to refer their positions to the same system of co-ordinates.

"3. By means of the provisional tables to calculate the apparent places of the planets for the epochs of the observations.

"4. To compare the observed with the tabular places; to deduce the corrections of the elliptic elements of the four planets, and to determine if the agreement is then perfect.

"5. Should the agreement not be satisfactory, to investigate the cause of the discordance."

Observations of the Planet Venus, with a 6-inch Silvered Glass Reflector. By R. Langdon, Esq.

(*Communicated by J. Norman Lockyer, F.R.S.*)

On May 1st, 1871, I had a good view of the planet *Venus*; but I could not at first see her to my satisfaction as her light was so bright. She had more the appearance of a miniature sun than a star; but I put a diaphragm of blackened card in the eye-piece, and made a small hole through its centre with a piece of hot wire. I found this arrangement keep out to a great extent the glaring rays. I also sometimes used a slip of slightly-tinted glass in